SAFE HANDS & IIT-ian's PACE

MONTHLY MAJOR TEST-01 (JEE) ANS KEY Dt. 26-07-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	D	31	C	61	B
2	C	32	C	62	C
3	D	33	A	63	A
4	D	34	B	64	A
5	B	35	B	65	C
6	A	36	B	66	A
7	D	37	C	67	A
8	D	38	A	68	D
9	A	39	A	69	B
10	D	40	D	70	C
11	A	41	C	71	B
12	B	42	B	72	D
13	A	43	B	73	B
14	D	44	A	74	A
15	D	45	A	75	A
16	D	46	D	76	D
17	C	47	C	77	A
18	D	48	C	78	C
19	C	49	A	79	D
20	C	50	A	80	A
21	8	51	5	81	4
22	2	52	95.2	82	8
23	2	53	16	83	2
24	5	54	29	84	3
25	6.5	55	0.2	85	8
26	7	56	10	86	6
27	3	57	2	87	5
28	5	58	6	88	42
29	0	59	12	89	1
30	2	60	5	90	7

SAFE HANDS & IIT-ian's PACE

MAJOR TEST-01 (NB-16 JEE) Physics Answer key & Solutions

: ANSWER KEY :

1)	d	2)	c	3)	d	4)	d	21)	8	22)	2	23)	2	24)	5
5)	b	6)	a	7)	d	8)	d	25)	6.5	26)	7	27)	3	28)	5
9)	a	10)	d	11)	a	12)	b	29)	0	30)	2				
13)	a	14)	d	15)	d	16)	d								
17)	c	18)	d	19)	c	20)	c								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

1 **(d)**

Man walks from his home to market with a speed of 5 km/h. Distance= 2.5 km and time $= \frac{d}{v} = \frac{2.5}{5} = \frac{1}{2}$ hr and he returns back with speed of 7.5 km/h in rest of time of 10 $minutes$

Distance $= 7.5 \times \frac{10}{60} = 1.25\ km$

So, Average speed $= \frac{\text{Total distance}}{\text{TOtal time}}$

$= \frac{(2.5+1.25)km}{(40/60)hr} = \frac{45}{8} km/hr$

2 **(c)**

$S_n = u + \frac{a}{2}(2n-1) \Rightarrow 1.2 = 0 + \frac{a}{2}(2 \times 6 - 1)$

$\Rightarrow a = \frac{1.2 \times 2}{11} = 0.218\ m/s^2$

3 **(d)**

An aeroplane flies 400m north and 300m south so the net displacement is 100m towards north.

Then it flies 1200m upward so $r = \sqrt{(100)^2 + (1200)^2}$

$= 1204 \text{ m} \simeq 1200 \text{ m}$

4 **(d)**

$\vec{v}_{man} = \frac{v}{\sqrt{2}}\hat{\imath} + \frac{v}{\sqrt{2}}\hat{\jmath}$

Let $\vec{v}_{wind} = a\hat{\imath} + b\hat{\jmath}$

$\Rightarrow \vec{v}_{wind/man} = \left(a - \frac{v}{\sqrt{2}}\right)\hat{\imath} + \left(b - \frac{v}{\sqrt{2}}\right)\hat{\jmath}$

$\Rightarrow \tan\theta = \frac{b - \frac{v}{\sqrt{2}}}{a - \frac{v}{\sqrt{2}}} = \tan 270°$

$\Rightarrow a - \frac{v}{\sqrt{2}} = 0$

$\Rightarrow a = \frac{v}{\sqrt{2}} \Rightarrow \vec{v}_{wind} = \frac{v}{\sqrt{2}}\hat{\imath} + b\hat{\jmath}$

when the man doubles his speed

$\vec{v}'_{man} = 2\left(\frac{v}{\sqrt{2}}\hat{\imath} + \frac{v}{\sqrt{2}}\hat{\jmath}\right) = \sqrt{2}\,(v\hat{\imath} + v\hat{\jmath})$

$\Rightarrow \vec{v}'_{wind/man} = \left(\frac{v}{\sqrt{2}} - \sqrt{2}\,v\right)\hat{\imath} + (b - \sqrt{2}\,v)\hat{\jmath}$

$\Rightarrow \tan\theta' = \frac{b - \sqrt{2}}{\frac{v}{\sqrt{2} - \sqrt{2}\,v}} = \frac{2v - \sqrt{2}\,b}{v}$

But $\theta' = 270° - \cot^{-1}(2)$

$\Rightarrow \tan[270° - \cot^{-1}(2)] = \frac{2v - \sqrt{2b}}{v}$

$\Rightarrow \cot[\cot^{-1}(2)] = \frac{2v - \sqrt{2b}}{v}$

$\Rightarrow 2v = 2v - \sqrt{2}\,b = b = 0$

$\therefore \vec{v}_{wind} = \frac{v}{\sqrt{2}}\hat{\imath}$

5 **(b)**

Distance covered =Area enclosed by $v - t$ graph

= Area of triangle $= \frac{1}{2} \times 4 \times 8 = 16\ m$

6 **(a)**

$x = \frac{1}{2}gt^2, 100 - x = 25x - \frac{1}{2}gt^2;$

Adding $25t = 100$ or $t = 4$ s

8 **(d)**

$f = \frac{uv}{u+v}, \frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{(u+v)}{u+v}$

9 **(a)**

By submitting the dimensions of each quantity we get

$T = [ML^{-1}T^{-2}]^a[L^{-3}M]^b[MT^{-2}]^c$

11 **(a)**

Given, $W = \frac{1}{2}kx^2$

Writing the dimensions on both sides

$[\text{ML}^2\text{T}^{-2}] = k[\text{M}^0\text{L}^2\text{T}^0]$

$\therefore$ Dimensions of $k = [\text{MT}^{-2}] = [\text{ML}^0\text{T}^{-2}]$

13 **(a)**

$[e] = [AT], \epsilon_0 = [M^{-1}L^{-3}T^4A^2], [h] = [ML^2T^{-1}]$

And $[c] = [LT^{-1}]$

$\therefore \left[\frac{e^2}{4\pi\epsilon_0 hc}\right] = \left[\frac{A^2T^2}{M^{-1}L^{-3}T^4A^2 \times ML^2T^{-1} \times LT^{-1}}\right]$

$= [M^0L^0T^0]$

14 **(d)**

Dimensional formula of magnetic flux

$= [\text{ML}^2\text{T}^{-2}\text{A}^{-1}]$

15 **(d)**

$\sqrt{P^2 + Q^2 + 2PQ\cos\theta} = (P - Q)$

$\Rightarrow P^2 + Q^2 + 2PQ\cos\theta = P^2 + Q^2 - 2PQ$

$\Rightarrow 2PQ(1+\cos\theta)=0$

but $2PQ \neq 0$

$\therefore 1 + \cos\theta = 0$ or $\cos\theta = -1$

or $\theta = 180°$

16 **(d)**

Let $\vec{\mathbf{u}}_1$ and $\vec{\mathbf{u}}_2$ be the initial velocities of the two particles and θ_1 and θ_2 be their angles of projection with the horizontal

The velocities of the two particles after time t are,

$\vec{\mathbf{v}}_1 = (u_1 \cos\theta_1)\hat{\mathbf{i}} + (u_1 \sin\theta_1 - gt)\hat{\mathbf{j}}$ and

$\vec{\mathbf{v}}_1 = (u_1 \cos\theta_1)\hat{\mathbf{i}} + (u_2 \sin\theta_2 - gt)\hat{\mathbf{j}}$

Their relative velocity is $\vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2$

$= (u_1 \cos\theta_1 - u_2 \cos\theta_1)\,\hat{\mathbf{i}} + (u_1 \sin\theta_1 - u_2 \sin\theta_2)\hat{\mathbf{j}}$

Which is a constant. So the path followed by one, as seen by the other is a straight line, making a constant angle with the horizontal

17 **(c)**

The pilot will see the ball falling in straight line because the reference frame is moving with the same horizontal velocity but the observer at rest will see the ball falling in parabolic path

18 **(d)**

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x v_y}{g}$$

$\therefore$ Range $\propto$ horizontal initial velocity (u_x)

In path 4 range is maximum so football possess maximum horizontal velocity in the path

19 **(c)**

The forces acting on the ball will be (i) in the direction opposite to its motion *ie*, frictional force and(ii) weight mg.

20 **(c)**

Change in velocity $= 2v\sin(\theta/2) = 2v\sin 20°$

Integer Answer Type

21 **(8)**

$$t_1 = t_2 - t, v_1 = v_2 = v, S = \frac{1}{2}a_1t_2^1, S = \frac{1}{2}a_2t_2^2$$

$v_1 = a_1t_1, v_2 = a_2t_2 \Rightarrow v_2 + v = a_1t_1$

$$\Rightarrow a_2t_2 + v = a_1t_1 = a_1t_2 \Rightarrow t_2 = \frac{v + a_1t}{a_1 - a_2}$$

$$\sqrt{\frac{a_2}{a_1}} = \frac{t_1}{t_2} = 1 - \frac{t}{t_2} \Rightarrow \sqrt{\frac{a_2}{a_1}} = 1 - \frac{t(a_1 - a_2)}{(v + a_1t)}$$

$$\Rightarrow \frac{\sqrt{a_2}}{\sqrt{a_1}} = \frac{v + a_2t}{v + a_1t} \Rightarrow \sqrt{a_2}\,v + a_1\sqrt{a_2}\,t = v\sqrt{a_1} + a_2\sqrt{a_1}t$$

$\Rightarrow v = \left(\sqrt{a_1a_2}\right)t = 8\ \mathrm{ms}^{-1}$

22 **(2)**

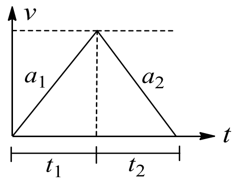
Taking upward direction as positive, let us work in the frame of lift. Acceleration of ball relative to lift $= (g + a)$ downward, so $a_{real} = -(g + a)$, initial velocity: $u_{rel} = v$, final velocity: $v_{rel} = -v$ as the ball will reach the man with same speed w.r.t lift

Apply $v_{rel} = u_{rel} + a_{rel}t \Rightarrow -v = v + (-g - a)t \Rightarrow t = 2$ s

23 **(2)**

$t_1 + t_2 = 4$ min, $v = a_1t_1 = a_2t_2$

$$S = \frac{1}{2} \times 4v \Rightarrow 4 = 2v \Rightarrow v = 2$$



$$t_1 + t_2 = v\left[\frac{1}{a_1} + \frac{1}{a_2}\right] \Rightarrow 4 = 2\left[\frac{1}{a_1} + \frac{1}{a_2}\right] \Rightarrow \frac{1}{a_1} + \frac{1}{a_2} = 2$$

24 **(5)**

$$s = u + \frac{a}{2}(2n - 1)$$

$u = 100\ \mathrm{ms}^{-1}, a = -10\ \mathrm{ms}^{-2}$ and $s = 5$ m

$5 = 100 - 5(2n - 1)$ gives $n = 10$ s

Body when thrown up with velocity $200\ \mathrm{ms}^{-1}$ will take 20 s to reach the highest point. Distance travelled in 20th second is $200 - 5(200 \times 2 - 1) = 5$ m

In the last second of upward journey, the bodies will travel same distance

25 **(6.5)**

Density $(\rho) = \frac{\text{Mass}}{\text{Volume}} = \frac{\mathrm{m}}{l^3}$ .....(for cube $V = l^3$)

Percentage relative error in density will be,

$$\frac{\Delta\rho}{\rho} \times 100 = \left[\frac{\Delta \mathrm{m}}{\mathrm{m}} + 3\frac{\Delta l}{l}\right]\%$$

$= [3.5 + (3 \times 1)]\%$

$= (3.5 + 3)\% = 6.5\%$

26 **(7)**

Dimension of force $= [\mathrm{M}^1\mathrm{L}^1\mathrm{T}^2]$

Now, $\mathrm{F} = \mathrm{ma}$

$$\Rightarrow \mathrm{m} = \frac{\mathrm{f}}{\mathrm{a}}$$

$$\therefore \text{Mass} = \frac{10^3\mathrm{N}}{10^{-2}\mathrm{m}\,(10^1\mathrm{s})^{-2}} = \frac{10^3\mathrm{kg\ m}}{\mathrm{s}^2} \times \frac{10^2 s^2}{10^{-2}\mathrm{m}} = 10^7\mathrm{kg}$$

Comparing 10^{x} with 10^7,

$\therefore \mathrm{x} = 7$

27 **(3)**

From Coulomb's law,

$$\mathrm{F} = \frac{\mathrm{q}_1\mathrm{q}_2}{4\pi\mathrm{F}r^2}$$

$\therefore\ \varepsilon_0 = \frac{q_1q_2}{4\pi Fr^2} = \frac{(A^1T^1)(A^1T^1)}{[M^1L^1T^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$

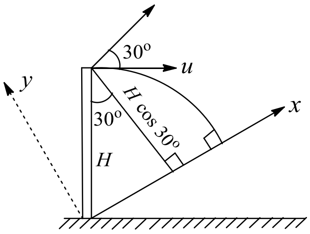
$\therefore$ p $= -1$, q $= -3, r = 4, s = 2$

$\therefore\ \frac{p=q+r}{s} = \frac{-1-(-3)+4}{2} = \frac{6}{2} = 3$

28 **(5)**

$v_x = u_x + a_x t \Rightarrow 0 = u\cos 30° - g\sin 30°\ t$

$\Rightarrow t = \frac{u\sqrt{3}}{g}$



$S_y = u_y t + \frac{1}{2}a_y t^2$

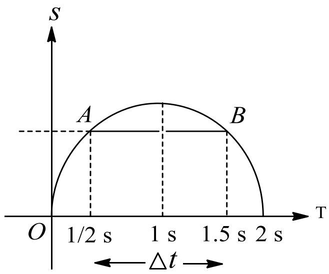
$\Rightarrow -H\cos 30° = -u\sin 30°\, t - \frac{1}{2}g\cos 30°\, t^2$

$\Rightarrow\ -H\frac{\sqrt{3}}{2} = \frac{-u}{2}\frac{u\sqrt{3}}{g} - \frac{1}{2}g\frac{\sqrt{3}}{2}\frac{u^2 3}{g^2}$

$\Rightarrow u = \sqrt{2gH/5} = \sqrt{2\times 10\times 6.25/5} = 5$ m/s

29 **(0)**

As $s - t$ graph is a parabola, it can be given as $s = k_1 t - k_2 t^2$, where k_1 and k_2 are constants. Since it is symmetrical $t = 1$ s, we have equal displacements at $t = \frac{1}{2}s$ and $t = \frac{3}{2}\ s$ that tells us that line AB joining two coordinates is parallel to the t-axis. Hence, slope of AB is zero. This implies that the average velocity during the time interval $\Delta t = 1$ s is zero



30 **(2)**

$AB = 2\,R\cos\theta$

$AB = \frac{1}{2}g\cos\theta t^2 \Rightarrow 2\,R\cos\theta = \frac{1}{2}g\cos\theta t^2$

$2\sqrt{\frac{R}{g}} = t \Rightarrow 2\sqrt{\frac{10}{10}} = t = 2s$

SAFE HANDS & IIT-ian's PACE

MAJOR TEST-01 (NB-16 JEE) Maths Answer key & Solutions

: ANSWER KEY :

61)	b	62)	c	63)	a	64)	a	81)	4	82)	8	83)	2	84)	3
65)	c	66)	a	67)	a	68)	d	85)	8	86)	6	87)	5	88)	42
69)	b	70)	c	71)	b	72)	d	89)	1	90)	7				
73)	b	74)	a	75)	a	76)	d								
77)	a	78)	c	79)	d	80)	a								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(b)**

We have,

$a = 5$ cm, $b = 4$ cm and $\cos(A-B) = \frac{31}{32}$

$\therefore \tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

$\Rightarrow \sqrt{\frac{1-\cos(A-B)}{1+\cos(A-B)}} = \frac{a-b}{a+b}\sqrt{\frac{1+\cos C}{1-\cos C}}$

$\Rightarrow \frac{1-\frac{31}{32}}{1+\frac{31}{32}} = \left(\frac{5-4}{5+4}\right)^2\left(\frac{1+\cos C}{1-\cos C}\right)$

$\Rightarrow \frac{81}{63} = \frac{1+\cos C}{1-\cos C} \Rightarrow \cos C = \frac{1}{8}$

62 **(c)**

We have, $(5+4\cos\theta)(2\cos\theta+1) = 0$

$\Rightarrow \cos\theta = -\frac{5}{4}$ which is not possible

$\therefore\ 2\cos\theta + 1 = 0 \ \Rightarrow\ \cos\theta = -\frac{1}{2}$

$\Rightarrow\ \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\therefore$ Solution set is $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \in [0, 2\pi]$

63 **(a)**

Given, $\cot(\alpha+\beta) = 0 \ \Rightarrow\ \cos(\alpha+\beta) = 0$

$\Rightarrow \alpha+\beta = (2n+1)\frac{\pi}{2}, n \in I$

$\therefore\ \sin(\alpha+2\beta) = \sin(2\alpha+2\beta-\alpha)$

$= \sin[(2n+1)\pi - \alpha]$

$= \sin(2n\pi + \pi - \alpha)$

$= \sin(\pi-\alpha) = \sin\alpha$

64 **(a)**

Since, $-1 \le \cos\theta \le 1$

$\therefore\ -1 \le \cos(4x-5) \le 1$

$\Rightarrow\ -3 \le 3\cos(4x-5) \le 3$

$\Rightarrow\ 4-3 \le 3\cos(4x-5)+4 \le 3+4$

$\Rightarrow\ 1 \le 3\cos(4x-5)+4 \le 7$

65 **(c)**

(1) $\cot\theta - \tan\theta = 2$

$\Rightarrow\ 2\cot 2\theta + 2 \ \Rightarrow\ \tan 2\theta = 1$

$\Rightarrow\ 2\theta = n\pi + \frac{\pi}{4} \ \Rightarrow\ \theta = (4n+1)\frac{\pi}{8}$

(2) The given equation can be written as

$2\sin x\cos x + 2\cos^2 x - 1 + \sin x + \cos x + 1 = 0$

$\Rightarrow\ (2\cos x + 1)(\sin x + \cos x) = 0$

$\Rightarrow\ \cos x = -\frac{1}{2}$ or $\sin x + \cos x = 0$

$\Rightarrow\ \cos x = -\frac{1}{2}$ or $\tan x = -1$

But $\cos x$ and $\tan x$ are positive in Ist quadrant. Therefore, the equation has no solution in the Ist quadrant. Hence, both of statements are correct.

66 **(a)**

Let $\sec\theta - \tan\theta = \lambda$ ...(i)

Then,

$(\sec\theta + \tan\theta) = \frac{1}{\sec\theta - \tan\theta}$

$\Rightarrow \sec\theta + \tan\theta = \frac{1}{\lambda}$ ...(ii)

$\therefore 2\tan\theta = \frac{1}{\lambda} + \lambda$ [On subtracting (i) from (ii)]

$\Rightarrow 2x - \frac{1}{2x} = \frac{1}{\lambda} - \lambda$

$\Rightarrow \lambda = \frac{1}{2x}, -2x \Rightarrow \sec\theta - \tan\theta = \frac{1}{2x}, -2x$

67 **(a)**

We have,

$\sum a^3\cos(B-C)$

$= \sum k^3\sin^3 A\cos(B-C)$

$= k^3\sum \sin^2 A\sin(B+C)\cos(B-C)$

$= \frac{k^3}{2}\sum \sin^2 A(\sin 2B + \sin 2C)$

$= \frac{k^3}{2}\sum[\sin^2 A(\sin 2B + \sin 2C)$
$+ \sin^2 B(\sin 2C + \sin 2A)$
$+ \sin^2 C(\sin 2A + \sin 2B)$

$= k^3\sum[\sin^2 A\sin B\cos B + \sin^2 B\sin A\cos A]$

$= k^3\sum \sin A\sin B\sin(A+B)$

$= k^3[\sin A\sin B\sin C + \sin B\sin C\sin A$
$+ \sin C\sin A\sin B]$

$= 3(k\sin A)(k\sin B)(k\sin C) = 3\,abc$

68 **(d)**

We have,

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
$= (\sec A + \tan A)(\sec B$
$+ \tan B)(\sec C + \tan C)$

$\Rightarrow (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C$
$- \tan^2 C)$

$= \{(\sec A + \tan A)(\sec B + \tan B)(\sec\ C$
$+ \tan C)\}^2$

$\Rightarrow 1 = \{(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)\}^2$

$\Rightarrow (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1$

Hence, LHS = RHS = ± 1

69 **(b)**

$\tan(A + B + C)$

$= \frac{[\tan A + \tan B + \tan C - \tan A \tan B \tan C]}{[1 - \tan A \tan B - \tan B \tan C - \tan C \tan A]}$

$\Rightarrow \tan(90°)$

$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$\Rightarrow \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

70 **(c)**

We have, $x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right)$

$= z \cos\left(\theta + \frac{4\pi}{3}\right) = k$ (say)

$\Rightarrow \cos\theta = \frac{k}{x}, \cos\left(\theta + \frac{2\pi}{3}\right) = \frac{k}{y}$

and $\cos\left(\theta + \frac{4\pi}{3}\right) = \frac{k}{z}$

$\therefore \frac{k}{x} + \frac{k}{y} + \frac{k}{z} = \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)$

$= \cos\theta - \cos\left(\frac{\pi}{3} - \theta\right) - \cos\left(\frac{\pi}{3} + \theta\right)$

$= \cos\theta - 2\cos\frac{\pi}{3}\cos\theta = 0$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

71 **(b)**

Given, $x = 3 + i$ ...(i)

Now, $x^3 - 3x^2 - 8x + 15$

$= (3 + i)^3 - 3(3 + i)^2 - 8(3 + i) + 15$

$= (27 + i^3 + 27i + 9i^2) - 3(9 + i^2 + 6i) - 24 - 8i + 15$

$= -15$

73 **(b)**

We have,

$x^2 - 2a_1 x + 1 = 0$ ...(i)

$x^2 - 4a_2 x + 2 = 0$ ...(ii)

$x^2 - 6a_3 x + 3 = 0$ ...(iii)

Let $\alpha, \beta; \beta, \gamma$ and γ, α be the pairs of roots of equations (i),(ii) and (iii) respectively. Then,

$\alpha + \beta = 2a_1, \alpha\beta = 1$ ...(iv)

$\beta + \gamma = 4a_2, \beta\gamma = 2$ ...(v)

$\gamma + \alpha = 6a_3, \gamma\alpha = 3$ ...(vi)

Now,

$\alpha\beta = 1, \beta\gamma = 2$ and $\gamma\alpha = 3$

$\Rightarrow (\alpha\beta)(\beta\gamma)(\gamma\alpha) = 1 \times 2 \times 3 \Rightarrow \alpha, \beta\gamma = \pm\sqrt{6}$

$\therefore \alpha = \pm\sqrt{\frac{3}{2}}, \beta = \pm\sqrt{\frac{2}{3}}, \gamma = \pm\sqrt{6}$

and,

$\alpha + \beta - 2a_1, \beta + \gamma = 4a_2, \gamma + \alpha = 6a_3$

$\Rightarrow \alpha + \beta + \gamma = a_1 + 2a_2 + 3a_3$

$\therefore \alpha = a_1 - 2a_2 + 3a_3, \beta = a_1 + 2a_2 - 3a_3, \gamma = -a_1 + 2a_2 + 3a_3$

Thus, we have the following sets of simultaneous linear equations:

$a_1 - 2a_2 + 3a_3 = \sqrt{\frac{3}{2}}$ $a_1 - 2a_2 + 3a_3 = -\sqrt{\frac{3}{2}}$

$a_1 + 2a_2 - 3a_3 = \sqrt{\frac{2}{3}}$ and, $a_1 + 2a_2 - 3a_3 = -\sqrt{\frac{2}{3}}$

$-a_1 + 2a_2 + 3a_3 = \sqrt{6}$ $-a_1 + 2a_2 + 3a_3 = -\sqrt{6}$

Hence, there are two triplets (a_1, a_2, a_3)

74 **(a)**

If $ax^3 + bx + c$ is divisible by $x^2 + bx + c$, then the remainder must be zero when $ax^3 + bx + c$ is divided by $x^2 + bx + c$

We have,

$ax^3 + bx + c = (x^2 + bx + c)(ax - ab) + \{x(b - ac + ab^2) + c - abc\}$

$\therefore$Remainder = 0

$\Rightarrow x(b - ac + ab^2) - c + abc = 0$ for all x

$\Rightarrow b - ac + ab^2 = 0$ and $-c + abc = 0$

$\Rightarrow b - ac + ab^2 = 0$ and $ab = 1$ $[\because c \neq 0]$

$\Rightarrow b - ac + a\left(\frac{1}{a}\right)^2 = 0$ $[\because ab = 1 \Rightarrow b = 1/a]$

$\Rightarrow ab - a^2c + 1 = 0$

$\Rightarrow a^2c - ab - 1 = 0$

$\Rightarrow a$ is a root of $x^2c - bx - 1 = 0$

75 **(a)**

Since, $z\bar{z}(z^2 + \bar{z}^2) = 350$

$\Rightarrow 2(x^2 + y^2)(x^2 - y^2) = 350$

$\Rightarrow (x^2 + y^2)(x^2 - y^2) = 175$

Since, $x, y \in I$, the only possible case which gives integral solution, is

$x^2 + y^2 = 25$ ...(i)

$x^2 - y^2 = 7$ ...(ii)

From Eqs. (i) and (ii), we get

$x^2 = 16, \quad y^2 = 9$

$\Rightarrow \quad x = \pm 4, \quad y = \pm 3$

$\therefore$ Area of rectangle $= 8 \times 6 = 48$

76 **(d)**

Given, $\log_{27} \log_3 x = \frac{1}{3}$

$\Rightarrow (\log_3 x) = (27)^{1/3} = 3$

$\Rightarrow x = (3)^3$

$\Rightarrow x = 27$

77 **(a)**

Let roots be α and 2α

$\therefore \quad \alpha + 2\alpha = 3\alpha = -\frac{(3a-1)}{(a^2-5a+3)}$

And $\alpha . 2\alpha = 2\alpha^2 = \frac{2}{(a^2-5a+3)}$

$\Rightarrow \frac{(3a-1)^2}{9(a^2-5a+3)^2} = \frac{1}{(a^2-5a+3)}$

$\Rightarrow (3a-1)^2 = 9(a^2-5a+3)$

$\Rightarrow 45a - 6a = 27 - 1 \quad \Rightarrow \quad a = \frac{2}{3}$

78 **(c)**

Since, n is not a multiple of 3, therefore $n = 3m + 1, n = 3m + 2$, where m is a positive integer.

For $n = 3m + 1$,

$1 + \omega^n + \omega^{2n} = 1 + \omega^{3m+1} + \omega^{2(3m+1)}$

$= 1 + \omega^{3m}\omega + (\omega^3)^{2m}\omega^2 = 1 + \omega + \omega^2 = 0$

Similarly, for $n = 3m + 2$

$\therefore \quad 1 + \omega^n + \omega^{2n} = 1 + \omega^{3m+2} + \omega^{2(3m+2)}$

$= 1 + \omega^{3m}.\omega^2 + (\omega^3)^{2m}.\omega^3.\omega = 0$

$[\because \ \omega^3 = 1]$

79 **(d)**

Let D_1 and D_2 be discriminants of $x^2 + b_1 x + c_1 = 0$ and $x^2 + b_2 x + c_2 = 0$ respectively. Then

$D_1 + D_2 = b_1^2 - 4c_1 + b_2^2 - 4c_2$

$= (b_1^2 + b_2^2) - 4(c_1 + c_2)$

$= b_1^2 + b_2^2 - 2b_1 b_2 \quad [\because b_1 b_2 = 2(c_1 + c_2)$, given]

$= (b_1 - b_2)^2 \geq 0$

$\Rightarrow D_1 \geq 0$ or $D_2 \geq 0$

$\Rightarrow D_1$ and D_2 both are positive.

80 **(a)**

Let $f(x) = ax^2 + bx + c$

If the roots of $f(x) = 0$ are imaginary, then we cannot say anything about b (i.e. it can be positive, negative or zero). So, options (b),(c) and (d) are not necessarily true

Further, if $a > 0$, then the graph of $y = f(x)$ is above x-axis and hence

$f(x) > 0$ for all $x \in R \Rightarrow f(0) > 0 \Rightarrow c > 0$

$\therefore ac > 0$

Similarly, if $a < 0$, then the graph of $y = f(x)$ is below x-axis and hence

$f(x) < 0$ for all $x \in R \Rightarrow f(0) < 0 \Rightarrow c < 0$

$\therefore ac > 0$

Integer Answer Type

81 **(4)**

$$f(\theta) = \frac{1 - \sin 2\theta + \cos 2\theta}{2\cos 2\theta}$$

$$= \frac{(\cos\theta - \sin\theta)^2 + (\cos^2\theta - \sin^2\theta)}{2(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$= \frac{\cos\theta}{\cos\theta + \sin\theta}$$

$$= \frac{1}{1 + \tan\theta}$$

$$f(11°) \cdot f(34°) = \frac{1}{(1 + \tan 11°)} \times \frac{1}{(1 + \tan 34°)}$$

$$= \frac{1}{(1 + \tan 11°)} \times \frac{1}{(1 + \tan(45° - 11°))}$$

$$= \frac{1}{(1 + \tan 11°)} \times \frac{1}{\left(1 + \frac{1 - \tan 11°}{1 + \tan 11°}\right)}$$

$$= \frac{1}{(1 + \tan 11°)} \times \frac{(1 + \tan 11°)}{2} = \frac{1}{2}$$

82 **(8)**

$(\log_{27} x^3)^2 = \log_{27} x^6$

$\Rightarrow (3\log_{27} x)^2 = 6\log_{27} x$

$\Rightarrow 3\log_{27} x\,(3\log_{27} x - 2) = 0$

$\Rightarrow x = 1$ or $\log_{27} x = \frac{2}{3}$

$\Rightarrow x = (27)^{2/3} = 9$

Difference $= 9 - 1 = 8$

83 **(2)**

$\cos 290° = \sin 20°$; $\sin 250° = -\sin 70°$
$= -\cos 20°$

$$\Rightarrow \frac{1}{\sin 20°} - \frac{1}{\sqrt{3}\cos 20°}$$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sqrt{3}\sin 20° \cos 20°}$$

$$= \frac{2[\sin 60° \cos 20° - \sin 20° \cos 60°]}{\sqrt{3}\sin 20° \cos 20°}$$

$$= \frac{4\sin 40°}{\sqrt{3}\sin 40°} = \frac{4\sqrt{3}}{3}$$

Hence, the greatest integer less than or equal to is 2

84 **(3)**
Given equations can be written as

$x\sin 3\theta = \frac{\cos 3\theta}{y} - \frac{\cos 3\theta}{z} = 0$
...(i)

$x\sin 3\theta = \frac{2\cos 3\theta}{y} - \frac{2\sin 3\theta}{z} = 0$
...(ii)

and $x\sin 3\theta - \frac{2}{y}\cos 3\theta - \frac{1}{z}(\cos 3\theta + \sin 3\theta) = 0$
...(iii)

Eqs. (ii) and (iii), implies

$2\sin 3\theta = \cos 3\theta + \sin 3\theta$

$\Longrightarrow \quad \sin 3\theta = \cos 3\theta$

$\therefore \quad \tan 3\theta = 1$

$\Longrightarrow \quad 3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$

Or $\quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$

85 **(8)**

$$s = \frac{26 + 28 + 30}{2} = 42$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42(42-26)(42-28)(42-30)}$$

$$= \sqrt{42 \times 16 \times 14 \times 12}$$

$$= \sqrt{(7\times 6)\times(4\times 4)\times(7\times 2)\times(6\times 2)}$$

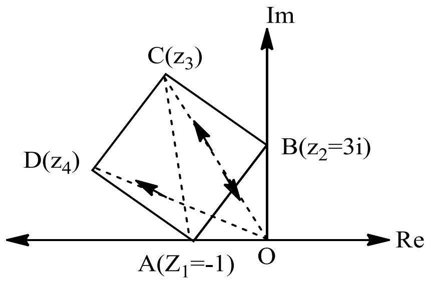
$$= 7 \times 6 \times 4 \times 2$$

$$r = \frac{\Delta}{s} = \frac{7\times 6\times 4\times 2}{42} = 8$$

86 **(6)**
Let the roots be $a-3d, a-d, a+d, a+3d$
Sum of roots $= 4a = 0 \Rightarrow a = 0$
Hence, roots are $-3d, -d, d, 3d$
Product of roots $= 9d^4 = m^2 \Rightarrow d^2 = \frac{m}{3}$ (1)
Again $\sum x_1x_2 = 3d^2 - 3d^2 - 9d^2 - d^2 - 3d^2 + 3d^2 = -10d^2$
$= -(3m+2)$
$\Rightarrow \frac{10m}{3} = 3m + 2 \Rightarrow 10m = 9m + 6$
$\Rightarrow m = 6$

87 **(5)**
$|3z + 9 - 7i| = |(3z + 6 - 3i) + (3 - 4i)|$
$\le |3z + 6 - 3i| + |3 - 4i|$
$= 3|z + 2 - i| + 5$
$= 20$

88 **(42)**



Vector AB represents $3i - (-1) = 3i + 1$
Vectors BC and AD represent $i(3i + 1) = i - 3$
Vector OC represents OB + BC
i.e. $3i + i - 3 = 4i - 3 = z_3$
Vector OD represents OA + AD
i.e $-1 + i - 3 = -4 + i = z_4$
$\Rightarrow |z_3|^2 + |z_4|^2 = 16 + 9 + 16 + 1 = 42$

89 **(1)**
$|\bar{z}w| = |\bar{z}||w|$
$= |z||w| = |zw|$
$1\ \arg(\bar{z}w) = \arg(\bar{z}) + \arg(w)$
$= \arg w - \arg z = -\frac{\pi}{2}\bar{z}w$
$= |\bar{z}w|e^{(\arg \bar{z}w)i} = 1 \cdot e^{\frac{-i\pi}{2}} = \cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right)$
$= -i$
$\Rightarrow a = 0, b = -1$

$\Rightarrow \mathrm{a}^2 + \mathrm{b}^2 = 1$

90 **(7)**

Given $a + b + c = 1$ (1)

$ab + bc + ca = 0$ (2)

$abc = 2$ (3)

Now $(a + b + c)^2 = 1$

$a^2 + b^2 + c^2 + 2\sum ab = 1$

$\therefore a^2 + b^2 + c^2 = 1$

Now, $a^3 + b^3 + c^3 - 3abc = (a + b + c)[\sum a^2 - \sum ab]$

$= 1(1 - 0) = 1$

$a^3 + b^3 + c^3 = 1 + 3abc = 1 + 3 \times 2 = 7$